

Moisture Conversion Calculations

A. Variables which are used to quantify the Actual Amount of Moisture

1. vapor pressure (e)

The pressure exerted by water vapor molecules.

Common atmospheric pressure (p) is measured as the sum of the air pressure and vapor pressure and $p \gg e$.

2. mixing ratio (w)

- The ratio of the mass of the water vapor to the mass of the dry air.

$$w = \frac{\text{MassOfWaterVapor}}{\text{MassOfDryAir}}$$

- Mixing ratio is a dimensionless quantities. However, since mass of water vapor is much less than the mass of dry air in typical atmospheric condition, mixing ratio is more commonly expressed in g/kg.

$$w = \epsilon \frac{e}{p - e}$$

where $\epsilon \approx 0.622$ is the ratio of the molecular weight of water vapor to dry air.

3. specific humidity (q)

The ratio of the mass of the water vapor to the mass of the total air.

$$q = \frac{\text{MassOfWaterVapor}}{\text{MassOfDryAir} + \text{MassOfWaterVapor}} \approx w$$
$$q = \frac{w}{1+w} \text{ and } w = \frac{q}{1-q}$$

4. absolute humidity or density (ρ)

The mass of the water vapor to the volume of the water vapor. It is also commonly referred as the density of the water vapor.

5. Dew point Temperature (T_d)

The temperature at which the amount of water vapor will become saturated.

B. Saturation

- When liquid water and water vapor reaches thermodynamic equilibrium, that is evaporation is the same as condensation, it is referred to as saturation.
- Saturation can be interpreted as the maximum amount of water vapor that can exist in a stable condition. Under normal circumstances, excessive water vapor will condense into liquid water. Supersaturation in typical atmospheric condition is possible but rare.
- The variables describing the saturation is the same as in (A) and all the equations in (A) remain the same as long as the subscript (s) is on both side of the equation.

1. saturation vapor pressure (e_s)

2. saturation mixing ratio (w_s)
 3. saturation specific humidity (q_s)
 4. saturation absolute humidity ρ_s
 5. temperature (T)
- Saturation of water vapor (e_s) is a function of temperature only and does not depend on the air. Equations of $e_s(T)$ can be found at <http://hurri.kean.edu/~yoh/calculations/satvap/satvap.html>
 - However, saturation mixing ratio (w_s or q_s) will depend on total atmospheric pressure (p) since a change of atmospheric pressure means a change of mass of the dry air, and mixing ratio depends on the mass of the dry air.

$$w_s = w_s(T, p) = \epsilon \frac{e_s}{p - e_s}$$

- **Temperature (T) describes Saturation (e_s).**

$$e_s = e_s(T) = 6.112 \exp\left(\frac{17.67T}{T + 243.5}\right)$$

Dew Point Temperature (T_d) describes the actual amount of water vapor (e), which is the saturation value at dew point temperature.

$$e = e_s(T_d) = 6.112 \exp\left(\frac{17.67T_d}{T_d + 243.5}\right)$$

Note that the form of the equations are the same (in this case, Bolton (1980) and temperature is measured in Celcius) in both cases.

C. Relative Humidity (Rh)

$$Rh = \frac{e}{e_s}$$

Relative humidity will only depend on temperature if vapor pressured is used in the definition. WMO has also adopted the definition

$$Rh = \frac{w}{w_s} \approx \frac{e}{e_s}$$

which will have a slight pressure dependence. The two definition only differ slightly in typical atmospheric condition.

D. Wet Bulb Temperature (T_w)

$$e = e_s(T_d) = e_s(T_w) - p \frac{c_p}{\epsilon L_v} (T - T_w)$$

where p is the total atmospheric pressure,

c_p is the specific heat capacity of dry air ($1005 \text{ J kg}^{-1} \text{ deg}^{-1}$)

L_v is the latent heat of vaporization ($2.5 \times 10^6 \text{ J kg}^{-1} \text{ deg}^{-1}$ at 0C)

- For any given pressure, this equation involves three variables (T , T_d and T_w). By given any two of them, the third one can be determined.

Fixed Point Properties of Water (according to International Temperature Scale of 1990, ITS-90)

- triple point temperature = 0.01 °C (273.16 K)
 - triple pressure = 6.1173 mb
 - triple point density of liquid = 0.99978 g / cm³
 - triple point density of vapor = 4.885 mg / L
 - boiling point = 99.975 °C at 1013.25 mb
 - melting point = 0 °C at 1013.25 mb
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Saturation Vapor Pressure over Liquid Water(T : Temperature in Kelvin)

- Goff-Gratch Equation

$$e_s = e_{st} 10^Z$$

where

$$Z = a\left(\frac{T_s}{T} - 1\right) + b \log_{10}\left(\frac{T_s}{T}\right) + c(10^{d(1-\frac{T}{T_s})} - 1) + f(10^{h(\frac{T_s}{T}-1)} - 1)$$

a = -7.90298	b = 5.02808	c = -1.3816 * 10 ⁻⁷
d = 11.344	f = 8.1328 * 10 ⁻³	h = -3.49149
e _{st} = 1013.246 mb	T _s = 373.16 K	

using boiling point at 373.16 K and saturation vapor pressure at boiling point at 1013.246mb (compared with ITS-90 values of boiling point at 373.15 K with saturation vapor pressure 1013.25 mb)

- 6th order Polynomial (Flatau et. al.,1992) (-50 to 50C)

$$e_s = a_1 + a_2*(T - 273.15) + ... + a_6*(T - 273.15)^6$$

a ₁	6.11176750
a ₂	0.443986062
a ₃	0.143053301E-01
a ₄	0.265027242E-03
a ₅	0.302246994E-05
a ₆	0.203886313E-07
a ₇	0.638780966E-10

- Bolton (1980) (0.3% within -35C to 35C)

$$e_s = 6.112 \exp\left(\frac{17.67 T_c}{T_c + 243.5}\right)$$

where T_c is the temperature in degree Celcius.

- Modified Clausius Clayperon Equation (0.7% @ -40C to 0.006% @ 40C)

$$\ln e_s = 53.67957 - \frac{6743.769}{T} - 4.8451 \ln T$$

- **Clausius Clayperon Equation (Latent Heat of Vaporization L_v is a function of T)**
(0.5% within -20C to 30 C)

$$L_v = L_{vo} + \Delta c(T - T_o)$$

$$\ln \frac{e_s}{e_o} = \frac{L_{vo} - (\Delta c)T_o}{R_v} \left(\frac{1}{T_o} - \frac{1}{T} \right) + \frac{\Delta c}{R_v} \ln \frac{T}{T_o}$$

$$\Delta c = (c_p)_v - (c_p)_l$$

$T_o = 273.16 \text{ K}$	$e_o = 6.1173 \text{ mb}$	$R_v = 461.50 \text{ J K}^{-1} \text{ kg}^{-1}$
$L_{vo} = 2.501 * 10^6 \text{ J K}^{-1}$	$c_{pv} = 1870 \text{ J K}^{-1} \text{ kg}^{-1}$	$c_{pl} = 4190 \text{ J K}^{-1} \text{ kg}^{-1}$

(or use the melting point of water at $T_o = 273.15 \text{ K}$ with $e_o = 6.107$ to 6.112 mb)

- **Clausius Clayperon Equation (constant Latent Heat of Vaporization L_v)**

$$e_s = e_o \exp \left(\frac{L_{vo}}{R_v} \left(\frac{1}{T_o} - \frac{1}{T} \right) \right)$$

$T_o = 273.16 \text{ K}$	$e_o = 6.1173 \text{ mb}$	$R_v = 461.50 \text{ J K}^{-1} \text{ kg}^{-1}$
$L_{vo} = 2.501 * 10^6 \text{ J K}^{-1} \text{ kg}^{-1}$		

(or use the melting point of water at $T_o = 273.15 \text{ K}$ with $e_o = 6.107$ to 6.112 mb)

Saturaton Vapor Pressure Over Ice (T : Temperature in Kelvin)

- Goff-Gratch Equation

$$e_{si} = e_o 10^X$$

where

$$X = a\left(\frac{T_o}{T} - 1\right) + b \log_{10}\left(\frac{T_o}{T}\right) + c\left(1 - \frac{T}{T_o}\right)$$

a = -9.09718	b = -3.56654	c = 0.876793
e _o = 6.1071 mb	T _o = 273.16 K	

(compared with ITS-90 values of triple point of 273.16 K with saturation vapor pressure of 6.11173 mb)

- 6th order Polynomial (Flatau et.al., 1992) (-50C to 0C)

$$e_s = a_1 + a_2*(T - 273.15) + \dots + a_6*(T - 273.15)^6$$

a ₁	6.10952665
a ₂	0.501948366
a ₃	0.186288989E-01
a ₄	0.403488906E-03
a ₅	0.539797852E-05
a ₆	0.420713632E-07
a ₇	0.147271071E-09

- Modified Clausius Clayperon Equation (0.14% within -80C to 0 C)

$$\ln e_{si} = 23.33086 - \frac{6111.72784}{T} + 0.15215 \ln T$$

- Clausius Clayperon Equation (Latent Heat of Vaporization L_v is a function of T)

$$L_s = L_{so} + \Delta c(T - T_o)$$

$$\ln \frac{e_{si}}{e_o} = \frac{L_{so} - (\Delta c)T_o}{R_v} \left(\frac{1}{T_o} - \frac{1}{T} \right) + \frac{\Delta c}{R_v} \ln \frac{T}{T_o}$$

$$\Delta c = (c_p)_v - (c_p)_s$$

$T_o = 273.16 \text{ K}$	$e_o = 6.1173 \text{ mb}$	$R_v = 461.50 \text{ J K}^{-1} \text{ kg}^{-1}$
$L_{so} = 2.834 * 10^6 \text{ J K}^{-1}$	$c_{pv} = 1870 \text{ J K}^{-1} \text{ kg}^{-1}$	$c_{ps} = 2106 \text{ J K}^{-1} \text{ kg}^{-1}$

(or use the melting point of water at $T_o = 273.15 \text{ K}$ with $e_o = 6.107$ to 6.112 mb)

- **Clausius Clayperon Equation**

$$e_{si} = e_o \exp\left(\frac{L_{so}}{R_v} \left(\frac{1}{T_o} - \frac{1}{T}\right)\right)$$

$T_o = 273.16 \text{ K}$	$e_o = 6.1173 \text{ mb}$	$R_v = 461.50 \text{ J K}^{-1} \text{ kg}^{-1}$
$L_{so} = 2.834 * 10^6 \text{ J K}^{-1} \text{ kg}^{-1}$		

(or use the melting point of water at $T_o = 273.15 \text{ K}$ with $e_o = 6.107$ to 6.112 mb)

References

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STANDARD ATMOSPHERE (TROPOSPHERE)

Standard Atmosphere (Troposphere)

Standard Atmosphere within the troposphere (below 11.0 km)

- The pressure at zero altitude is $p_o = 1013.250$ mb.
- The temperature at zero pressure-altitude is $T_o = 288.15$ K

- The lapse rate of temperature $\Gamma = - \frac{dT}{dz} = 6.5^\circ\text{C} / \text{km}$ is constant.

$$T(z) = T_o - \Gamma z$$

- The standard atmosphere satisfies ideal gas law

$$p = \rho R_d T$$

where ρ is the density of the air and R_d is the gas constant for dry air ($= 287.04 \text{ J kg}^{-1} \text{ K}^{-1}$).

- The standard atmosphere is hydrostatic

$$dp = - \rho g dz$$

where g is the acceleration due to gravity and is constant ($= 9.80665 \text{ m s}^{-2}$) if z is measured in geopotential height.

Pressure-Height Relationship in Standard Atmosphere

Combining the hydrostatic equation and the ideal gas law,

$$dp = - \rho g dz = - \frac{p g}{R_d T} dz = - p \frac{g}{R_d (T_o - \Gamma z)}$$

pressure-height (p, z) relationship can be found by integrating the above equation :

$$\frac{p}{p_o} = \left(\frac{T_o - \Gamma z}{T_o} \right)^{\frac{g}{R_d \Gamma}} = \left(\frac{T}{T_o} \right)^{\frac{g}{R_d \Gamma}}$$

$$z = \frac{T_o}{\Gamma} \left[1 - \left(\frac{p}{p_o} \right)^{\frac{R_d \Gamma}{g}} \right]$$

Density-Height Relationship in Standard Atmosphere

Starting from the pressure-height relationship above and replacing pressure with density using the ideal gas law, we get :

$$\frac{p}{p_o} = \left(\frac{T}{T_o} \right)^{\frac{g}{R_d \Gamma}} = \frac{\rho R_d T}{\rho_o R_d T_o}$$

$$\frac{\rho}{\rho_o} = \left(\frac{T}{T_o} \right)^{\frac{g}{R_d \Gamma} - 1}$$

$$z = \frac{T_o}{\Gamma} \left[1 - \left(\frac{\rho}{\rho_o} \right)^{\frac{1}{\frac{g}{R_d \Gamma} - 1}} \right]$$

Pressure Altitude (also

see <http://hurri.kean.edu/~yoh/calculations/altimeter>)

Pressure altitude measures the altitude which corresponds to a given value of atmospheric pressure according to the standard atmosphere.

Input : pressure reading

Output : the height will be the pressure altitude

Density Altitude (also

see <http://hurri.kean.edu/~yoh/calculations/altimeter>)

Density altitude measures the altitude which corresponds to a given value of atmospheric density according to the standard atmosphere. Water vapor is neglected here.

Input : density reading (calculated from pressure and temperature using ideal gas law,

see <http://hurri.kean.edu/~yoh/calculations/idealgas>)

Output : the height will be the density altitude